BEE core - Order Representation	
Declaring Variables	(Booleans and Integers)
(1) $\texttt{new\_bool}(\texttt{X})$	declare Boolean $\texttt{X}$
(2) $\texttt{new\_int}(\texttt{I}, \texttt{c}_1, \texttt{c}_2)$	declare (order) $\texttt{I}$, $\texttt{c}_1 \leq \texttt{I} \leq \texttt{c}_2$
(2b) $\texttt{new\_int}(\texttt{I}, \texttt{D})$	declare (order) $\texttt{I}$, $\texttt{member}(\texttt{I}, \texttt{D})$
(2c) $\texttt{new\_int\_plusK}(\texttt{I}, \texttt{I}_1, \texttt{c})$	declare (order) $\texttt{I}$, $\texttt{I}_1 + \texttt{c} = \texttt{I}$
(2d) $\texttt{new\_int\_mulK}(\texttt{I}, \texttt{I}_1, \texttt{c})$	declare (order) $\texttt{I}$, $\texttt{I}_1 * \texttt{c} = \texttt{I}$
(2e) $\texttt{new\_int\_divK}(\texttt{I}, \texttt{I}_1, \texttt{c})$	declare (order) $\texttt{I}$, $\texttt{I}_1 \ / \ \texttt{c} = \texttt{I}$
(3) $\texttt{bool2int}(\texttt{X}, \texttt{I})$	$(\texttt{X} \Leftrightarrow \texttt{I} = 1) \wedge (\neg\texttt{X} \Leftrightarrow \texttt{I} = 0)$
(3b) $\texttt{int\_order2bool\_array}(\texttt{I}, \texttt{Xs}, \texttt{c}_{\texttt{min}})$	$\bigwedge_{0 \leq \texttt{i} < \lvert\texttt{Xs}\rvert} \texttt{Xs}[\texttt{i}] \Leftrightarrow (\texttt{I} \geq \texttt{i} + \texttt{C}_{\texttt{min}})$
Boolean (reified) statements	$\texttt{op} \in \{\texttt{or}, \texttt{and}, \texttt{xor}, \texttt{iff}\}$
(4) $\texttt{bool\_eq}(\texttt{X}_1, \texttt{X}_2)$ or $\texttt{bool\_eq}(\texttt{X}_1, -\texttt{X}_2)$	$\texttt{X}_1 = \texttt{X}_2$ or $\texttt{X}_1 = -\texttt{X}_2$
(5) $\texttt{bool\_array\_eq\_reif}([\texttt{X}_1, \ldots, \texttt{X}_n], \texttt{X})$	$(\bigwedge_{\texttt{i}<\texttt{j}} \texttt{X}_\texttt{i} = \texttt{X}_\texttt{j}) \Leftrightarrow \texttt{X}$
(6) $\texttt{bool\_array\_op}([\texttt{X}_1, \ldots, \texttt{X}_n])$	$\texttt{X}_1 \ \texttt{op} \ \texttt{X}_2 \cdots \texttt{op} \ \texttt{X}_n$
(7) $\texttt{bool\_array\_op\_reif}([\texttt{X}_1, \ldots, \texttt{X}_n], \texttt{X})$	$\texttt{X}_1 \ \texttt{op} \ \texttt{X}_2 \cdots \texttt{op} \ \texttt{X}_n \Leftrightarrow \texttt{X}$
(8) $\texttt{bool\_op\_reif}(\texttt{X}_1, \texttt{X}_2, \texttt{X})$	$\texttt{X}_1 \ \texttt{op} \ \texttt{X}_2 \Leftrightarrow \texttt{X}$
(9) $\texttt{bool\_ite}(\texttt{X}_1, \texttt{X}_2, \texttt{X}_3)$	$\texttt{X}_1 \ ? \ \texttt{X}_2 : \texttt{X}_3$
(10) $\texttt{bool\_ite\_reif}(\texttt{X}_1, \texttt{X}_2, \texttt{X}_3, \texttt{X})$	$(\texttt{X}_1 \ ? \ \texttt{X}_2 : \texttt{X}_3) \Leftrightarrow \texttt{X}$
Integer relations (reified)	$\texttt{rel} \in \{\texttt{leq}, \texttt{geq}, \texttt{eq}, \texttt{lt}, \texttt{gt}, \texttt{neq}\}$
and arithmetic	$\texttt{op} \in \{\texttt{plus}, \texttt{times}, \texttt{div}, \texttt{mod}, \texttt{max}, \texttt{min}\}$, $\texttt{op}' \in \{\texttt{plus}, \texttt{times}, \texttt{max}, \texttt{min}\}$
(11) $\texttt{int\_rel}(\texttt{I}_1, \texttt{I}_2)$	$\texttt{I}_1 \ \texttt{rel} \ \texttt{I}_2$
(12) $\texttt{int\_rel\_reif}(\texttt{I}_1, \texttt{I}_2, \texttt{X})$	$\texttt{I}_1 \ \texttt{rel} \ \texttt{I}_2 \Leftrightarrow \texttt{X}$
(13) $\texttt{int\_array\_allDiff}([\texttt{I}_1, \ldots, \texttt{I}_n])$	$\bigwedge_{\texttt{i}<\texttt{j}} \texttt{I}_\texttt{i} \neq \texttt{I}_\texttt{j}$
(14) $\texttt{int\_array\_allDiff\_reif}([\texttt{I}_1, \ldots, \texttt{I}_n], \texttt{X})$	$(\bigwedge_{\texttt{i}<\texttt{j}} \texttt{I}_\texttt{i} \neq \texttt{I}_\texttt{j}) \Leftrightarrow \texttt{X}$
(15) $\texttt{int\_array\_allDiffCond}([\texttt{I}_1, \ldots, \texttt{I}_n], [\texttt{X}_1, \ldots, \texttt{X}_n])$	$\bigwedge_{\texttt{i}<\texttt{j}} (\texttt{X}_\texttt{i} \wedge \texttt{X}_\texttt{j} \Rightarrow \texttt{I}_\texttt{i} \neq \texttt{I}_\texttt{j})$
(16) $\texttt{int\_abs}(\texttt{I}_1, \texttt{I})$	$\lvert\texttt{I}_1\rvert = \texttt{I}$
(17) $\texttt{int\_op}(\texttt{I}_1, \texttt{I}_2, \texttt{I})$	$\texttt{I}_1 \ \texttt{op} \ \texttt{I}_2 = \texttt{I}$
(18) $\texttt{int\_array\_op}'([\texttt{I}_1, \ldots, \texttt{I}_n], \texttt{I})$	$\texttt{I}_1 \ \texttt{op}' \cdots \texttt{op}' \ \texttt{I}_n = \texttt{I}$
Cardinality	$\texttt{rel} \in \{\texttt{leq}, \texttt{geq}, \texttt{eq}, \texttt{lt}, \texttt{gt}\}$
(19) $\texttt{bool\_array\_sum\_rel}([\texttt{X}_1, \ldots, \texttt{X}_n], \texttt{I})$	$(\Sigma \ \texttt{X}_\texttt{i}) \ \texttt{rel} \ \texttt{I}$
(20) $\texttt{bool\_array\_pb\_rel}([\texttt{c}_1, \ldots, \texttt{c}_n], [\texttt{X}_1, \ldots, \texttt{X}_n], \texttt{I})$	$(\Sigma \ \texttt{c}_\texttt{i} * \texttt{X}_\texttt{i}) \ \texttt{rel} \ \texttt{I}$
(21) $\texttt{int\_array\_sum\_rel}([\texttt{I}_1, \ldots, \texttt{I}_n], \texttt{I})$	$(\Sigma \ \texttt{I}_\texttt{i}) \ \texttt{rel} \ \texttt{I}$
(22) $\texttt{int\_array\_lin\_rel}([\texttt{c}_1, \ldots, \texttt{c}_n], [\texttt{I}_1, \ldots, \texttt{I}_n], \texttt{I})$	$(\Sigma \ \texttt{c}_\texttt{i} * \texttt{I}_\texttt{i}) \ \texttt{rel} \ \texttt{I}$
(23) $\texttt{int\_array\_sumCond\_rel}([\texttt{X}_1, \ldots, \texttt{X}_n], [\texttt{I}_1, \ldots, \texttt{I}_n], \texttt{I})$	$(\Sigma \ \texttt{X}_\texttt{i} * \texttt{I}_\texttt{i}) \ \texttt{rel} \ \texttt{I}$

Cardinality Module K	
only support non-negative integers	
(24) bool_array_sum_modK($[\mathtt{X}_1,\ldots,\mathtt{X}_n],\mathtt{c},\ \mathtt{I}$)	$((\Sigma\ \mathtt{X}_i)\ \mathtt{mod}\ \mathtt{c}) =\ \mathtt{I}$
(25) bool_array_sum_divK($[\mathtt{X}_1,\ldots,\mathtt{X}_n],\mathtt{c},\ \mathtt{I}$)	$((\Sigma\ \mathtt{X}_i)\ /\ \mathtt{c}) =\ \mathtt{I}$
(26) bool_array_sum_divModK($[\mathtt{X}_1,\ldots,\mathtt{X}_n],\mathtt{c},\ \mathtt{I}_d,\ \mathtt{I}_m$)	$((\Sigma\ \mathtt{X}_i)\ \mathtt{mod}\ \mathtt{c}) =\ \mathtt{I}_d$ $((\Sigma\ \mathtt{X}_i)\ /\ \mathtt{c}) =\ \mathtt{I}_m$
(27) int_array_sum_modK($[\mathtt{I}_1,\ldots,\mathtt{I}_n],\mathtt{c},\ \mathtt{I}$)	$((\Sigma\ \mathtt{I}_i)\ \mathtt{mod}\ \mathtt{c}) =\ \mathtt{I}$
(28) int_array_sum_divK($[\mathtt{I}_1,\ldots,\mathtt{I}_n],\mathtt{c},\ \mathtt{I}$)	$((\Sigma\ \mathtt{I}_i)\ /\ \mathtt{c}) =\ \mathtt{I}$
(29) int_array_sum_divModK($[\mathtt{I}_1,\ldots,\mathtt{I}_n],\mathtt{c},\ \mathtt{I}_d,\ \mathtt{I}_m$)	$((\Sigma\ \mathtt{I}_i)\ /\ \mathtt{c}) =\ \mathtt{I}_d$ $((\Sigma\ \mathtt{I}_i)\ \mathtt{mod}\ \mathtt{c}) =\ \mathtt{I}_m$
Boolean arrays relations (reified)	$\mathtt{rel} \in \{\mathtt{eq},\mathtt{neq}\}$
if the arrays are of different length then pad shorter with zeros	
(30) bool_arrays_rel($\mathtt{Xs}_1,\mathtt{Xs}_2$)	$\mathtt{Xs}_1\ \mathtt{rel}\ \mathtt{Xs}_2$
(31) bool_arrays_rel_reif($\mathtt{Xs}_1,\mathtt{Xs}_2,\mathtt{X}$)	$\mathtt{Xs}_1\ \mathtt{rel}\ \mathtt{Xs}_2 \Leftrightarrow \mathtt{X}$
(32) bool_arrays_lex($\mathtt{Xs}_1,\mathtt{Xs}_2$)	$\mathtt{Xs}_1 \preceq \mathtt{Xs}_2$
(33) bool_arrays_lexLt($\mathtt{Xs}_1,\mathtt{Xs}_2$)	$\mathtt{Xs}_1 \prec \mathtt{Xs}_2$
(34) bool_arrays_lex_reif($\mathtt{Xs}_1,\mathtt{Xs}_2,\mathtt{X}$)	$\mathtt{Xs}_1 \preceq \mathtt{Xs}_2 \Leftrightarrow \mathtt{X}$
(35) bool_arrays_lexLt_reif($\mathtt{Xs}_1,\mathtt{Xs}_2,\mathtt{X}$)	$\mathtt{Xs}_1 \prec \mathtt{Xs}_2 \Leftrightarrow \mathtt{X}$
Integer arrays relations (reified)	$\mathtt{rel} \in \{\mathtt{eq},\mathtt{neq}\}$
(36) int_arrays_rel($\mathtt{Is}_1,\mathtt{Is}_2$)	$\mathtt{Is}_1\ \mathtt{rel}\ \mathtt{Is}_2$
(37) int_arrays_rel_reif($\mathtt{Is}_1,\mathtt{Is}_2,\mathtt{X}$)	$\mathtt{Is}_1\ \mathtt{rel}\ \mathtt{Is}_2 \Leftrightarrow \mathtt{X}$
(38) int_arrays_lex($\mathtt{Is}_1,\mathtt{Is}_2$)	$\mathtt{Is}_1 \preceq \mathtt{Is}_2$
(39) int_arrays_lexLt($\mathtt{Is}_1,\mathtt{Is}_2$)	$\mathtt{Is}_1 \prec \mathtt{Is}_2$
(40) int_arrays_lex_implied($\mathtt{Is}_1,\mathtt{Is}_2,\mathtt{X}$)	$\mathtt{X} \Rightarrow \mathtt{Is}_1 \preceq \mathtt{Is}_2$
(41) int_arrays_lexLt_implied($\mathtt{Is}_1,\mathtt{Is}_2,\mathtt{X}$)	$\mathtt{X} \Rightarrow \mathtt{Is}_1 \prec \mathtt{Is}_2$
(42) int_arrays_lex_reif($\mathtt{Is}_1,\mathtt{Is}_2,\mathtt{X}$)	$\mathtt{Is}_1 \preceq \mathtt{Is}_2 \Leftrightarrow \mathtt{X}$
(43) int_arrays_lexLt_reif($\mathtt{Is}_1,\mathtt{Is}_2,\mathtt{X}$)	$\mathtt{Is}_1 \prec \mathtt{Is}_2 \Leftrightarrow \mathtt{X}$

Table 1: Syntax of BEE Constraints.

BEE core - Direct Representation	
Declaring Variables	
(1) $\texttt{new\_int\_direct}(\texttt{I}, \texttt{c}_1, \texttt{c}_2)$	declare (direct) I, $\texttt{c}_1 \leq \texttt{I} \leq \texttt{c}_2$
(1b) $\texttt{new\_int\_direct}(\texttt{I}, \texttt{D})$	declare (direct) I, $\texttt{member}(\texttt{I}, \texttt{D})$
(1c) $\texttt{new\_int\_direct\_plusK}(\texttt{I}, \texttt{I}_1, \texttt{c})$	declare (direct) I, $\texttt{I}_1 + \texttt{c} = \texttt{I}$
(2) $\texttt{int\_direct2bool\_array}(\texttt{I}, \texttt{Xs}, \texttt{c}_{\texttt{min}})$	$\bigwedge_{0 \leq \texttt{i} < \lvert\texttt{Xs}\rvert} \texttt{Xs}[\texttt{i}] \Leftrightarrow (\texttt{I} = \texttt{i} + \texttt{C}_{\texttt{min}})$
Integer relations (reified)	$\texttt{rel} \in \{\texttt{leq}, \texttt{geq}, \texttt{eq}, \texttt{lt}, \texttt{gt}, \texttt{neq}\}$
(3) $\texttt{direct\_rel}(\texttt{I}_1, \texttt{I}_2)$	$\texttt{I}_1$ rel $\texttt{I}_2$
(4) $\texttt{direct\_array\_allDiff}([\texttt{I}_1, \ldots, \texttt{I}_n])$	$\bigwedge_{\texttt{i}<\texttt{j}} \texttt{I}_\texttt{i} \neq \texttt{I}_\texttt{j}$

Table 2: Syntax of Direct BEE Constraints.

BEE core - Mix Representation	
BEE integers can have multi bit-level representations, and can be defined using one of the following: new_int (order), new_direct (direct), new_int_dual (order & direct).	
While most BEE constraints are applied only on the Order encoding of the integer (Table 1), few applied only on the Direct encoding (Table 2), and other few applied on both (e.g. int_array_allDiff).	
When a constraint is applied on an integer and the integer doesn't have (all) the requested representation(s) BEE will add the required representation to the integer and add a channeling constraint automatically.	
Declaring Mix Variables	
(1) $\texttt{new\_int\_dual}(\texttt{I}, \texttt{c}_1, \texttt{c}_2)$	declare (order & direct) I, $\texttt{c}_1 \leq \texttt{I} \leq \texttt{c}_2$
(1b) $\texttt{new\_int\_dual\_plusK}(\texttt{I}, \texttt{I}_1, \texttt{c})$	declare (order & direct) I, $\texttt{I}_1 + \texttt{c} = \texttt{I}$
Channeling Variables	
(2) $\texttt{channel\_int2direct}(\texttt{I})$	Adds Direct representation to an Order integer.
(3) $\texttt{channel\_direct2int}(\texttt{I})$	Adds Order representation to a Direct integer.

Table 3: Syntax of Mix BEE Constraints.